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ON ACOUSTIC WAVE DIFFRACTION BY PLATES CONNECTED AT A RIGHT ANGLE

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Two-dimensional stationary acoustic processes within a fluid-filled infinite domain bounded by the sides of a right angle are considered. The pressure for which the Helmholtz equation is assumed satisfied within the domain, and some conditions containing high order derivatives on the boundary are regarded as the desired quantities. Expressions for the boundary operators are not made specific. An exact representation is found for the pressure when the sound field is excited by a point type source in the fluid. A number of specific problems of hydroacoustic wave diffraction by two mutually perpendicular plates is examined.

1. Formulation of the problem. Let us seek the solution of the Helmholtz equation

$$(\Delta + k^2) P(x, y) = -\delta(x - x_0, y - y_0) \tag{1.1}$$

satisfying the boundary conditions

$$L_1 P(x, 0) = 0, \quad x > 0 \qquad L_2 P(0, y) = 0, \quad y > 0 \tag{1.2}$$

in the first quadrant of a Cartesian x, y coordinate system. Here $P(x, y)$ is the pressure in the fluid and k is the wave number. It is assumed that the time dependence is given by the factor $e^{-i\omega t}$ which is omitted throughout. The effect of the boundary operators is determined by the formulas

$$L_1 = m_{11}\left(-\frac{\partial^2}{\partial x^2}\right)\frac{\partial}{\partial y} + m_{12}\left(-\frac{\partial^2}{\partial x^2}\right) \qquad L_2 = m_{21}\left(-\frac{\partial^2}{\partial y^2}\right)\frac{\partial}{\partial x} + m_{22}\left(-\frac{\partial^2}{\partial y^2}\right) \tag{1.3}$$

where $m_{\alpha\gamma}$ $(\alpha, \gamma = 1, 2)$ are polynomials of their arguments. The coefficients of these polynomials are independent of the space variables x and y.

It is assumed that

1) The polynomial $m_2(\lambda)$ and the algebraic function $l_1(\lambda)$ and $m_1(\lambda), l_2(\lambda)$, respectively

$$m_\alpha(\lambda) = m_{\alpha 1}(\lambda^2 - k^2) i\lambda + m_{\alpha 2}(\lambda^2 - k^2)$$

$$l_\alpha(\lambda) = -m_{\alpha 1}(\lambda^2)\sqrt{\lambda^2 - k^2} + m_{\alpha 2}(\lambda^2) \quad (\alpha = 1, 2) \tag{1.4}$$

have no common roots. Here the branches of the radical $\sqrt{\lambda^2 - k^2}$ are selected in the customary manner. The slit at the point $\lambda = k$ is located entirely in the upper half-plane, while the slit from the point $\lambda = -k$ is drawn keeping the central symmetry of the sketch relative to the origin. It is considered that $\lim \operatorname{Re}\sqrt{\lambda^2 - k^2} = +\infty$ as $\lambda \to \pm\infty$ on the real axis of the main sheet of the Riemann surface.

2) The algebraic function $l_\alpha(\lambda)$ has no real roots for $\operatorname{Im} k > 0$.

The property (1) will be used below to construct the solution.

The property (2) is needed to satisfy the ultimate absorption principle [1].

The solution is sought in the class of continuous functions at the origin. Compliance with the principle of ultimate absorption is also assumed. In other words, in the presence of the positive imaginary part in the wave number $(k = k' + ik''; \ k', \ k'' > 0)$ the solution should decrease exponentially at infinity, and the case of real positive k is considered by means of the passage to the limit $k'' \to +0$.

The solution of the problem posed is easily obtained by the mapping method in the particular case of the simplest Dirichlet boundary conditions (free fluid surface $L_1 = 1$, $L_2 = 1$) or Neumann boundary conditions (rigid boundary

$$L_1 = \partial / \partial y, \ L_2 = \partial / \partial x)$$

The case of mixed or impedance boundary conditions ($m_{\alpha\gamma}$ are nonzero constants) can be considered by the method proposed by Buchwald in solving a diffraction problem for waves on a fluid surface [2]. The Buchwald method is extended below to the case of boundary-contact problems when the differential order of at least one of the operators L_α $(\alpha = 1, 2)$ exceeds unity. The presence of high order differential operators in the boundary conditions results in the fact that the solution of the problem posed at the beginning of Sect. 1 ceases to be unique. This solution, called general, contains some number N of arbitrary constants. As in the case examined in [3], this number can be determined from the formula (*)

$$N = E\left(\frac{N_1 + N_2 - 1}{2}\right) \tag{1.5}$$

Here $N_{1,2}$ are the orders of the differential operators $L_{1,2}$, and $E(x)$ denotes the integer part of x. Arbitrariness in the solution is eliminated after the addition of N independent boundary-contact conditions in the formulation of the problem. These conditions are the following in the case under consideration

$$\begin{gathered} R_{1\beta}P(0, 0) + R_{2\beta}P(0, 0) = 0 \qquad (\beta = 1, 2, \ldots N) \\ R_{1\beta}P(0,0) = \lim_{x\to 0}\left[s_{1\beta 1}\left(-i\frac{\partial}{\partial x}\right)\frac{\partial}{\partial y} + s_{1\beta 2}\left(-i\frac{\partial}{\partial x}\right)\right]P(x, 0) \\ R_{2\beta}P(0,0) = \lim_{y\to 0}\left[s_{2\beta 1}\left(-i\frac{\partial}{\partial y}\right)\frac{\partial}{\partial x} + s_{2\beta 2}\left(-i\frac{\partial}{\partial y}\right)\right]P(0, y) \end{gathered} \tag{1.6}$$

where $s_{\alpha\beta\gamma}$ $(\alpha, \ \gamma = 1, \ 2, \ \beta = 1, \ 2, \ \ldots N)$ are polynomials of their arguments.

Example 1. Diffraction at the right angle. Let us consider a fluid-filled tank $(x > 0, \ y > 0)$ whose walls $(x = 0, \ y > 0$ and $x > 0, \ y = 0)$ are elastic plates capable only of bending motions. In this case the following expressions hold for the boundary operators:

$$L_1 = \left(\frac{\partial^4}{\partial x^4} - k_1^4\right)\frac{\partial}{\partial y} + \nu_1 \tag{1.7}$$

$$L_2 = \left(\frac{\partial^4}{\partial y^4} - k_2^4\right)\frac{\partial}{\partial x} + \nu_2, \qquad \nu_\alpha = \frac{\rho\omega^2}{D_\alpha}$$

where k_α are the wave numbers of the bending waves in the plates, ρ is the fluid density, and D_α are the cylindrical plate stiffnesses $(\alpha = 1,2)$.

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*) Let us note that there is a misprint in (2.20) in [3]. This formula should be analogous to (1.5).

According to (1.5), the general solution of such a problem contains four arbitrary constants. To complete the description of this mechanical model it is necessary to indicate the mode on the plate junction (weld, hinge connection, crack). Let us concider the plates welded rigidly, which yields the following set of boundary-contact conditions:

$$\lim_{x\to 0}\frac{\partial P(x,0)}{\partial y}=0,\qquad \lim_{y\to 0}\frac{\partial P(0,y)}{\partial x}=0$$
$$\lim_{x\to 0}\frac{\partial^2 P(x,0)}{\partial x\,\partial y}+\lim_{y\to 0}\frac{\partial^2 P(0,y)}{\partial x\,\partial y}=0 \tag{1.8}$$
$$D_1\lim_{x\to 0}\frac{\partial^3 P(x,0)}{\partial x^2\,\partial y}+D_2\lim_{y\to 0}\frac{\partial^3 P(0,y)}{\partial y^2\,\partial x}=0$$

The first two conditions denote the absence of plate displacements at the origin. This circumstance is associated with the fact that each plate is assumed to possess infinite stiffness relative to longitudinal displacements. The last two conditions express the invariability of the angle between the plates and the absence of a secondary torque on the plate junction, respectively.

Example 2. Diffraction by plates connected in a T. Let two right angles formed by an infinite ($y=0$) and semi-infinite ($x=0,\ y>0$) plate welded at a right angle be filled with fluid. We consider the source field to be at a point in the first quadrant, and the elastic properties of the plates to be the same as in Example 1. The problem is to solve the Helmholtz equation (1.1), which is inhomogeneous for $x>0$ and homogeneous for $x<0$, under the boundary conditions

$$\left[\left(\frac{\partial^4}{\partial x^2}-k_1^4\right)\frac{\partial}{\partial y}+\nu_1\right]P(x,0)=0,\qquad x\neq 0 \tag{1.9}$$

and the conjugate conditions

$$\frac{1}{2}\left(\frac{\partial^4}{\partial y^4}-k_2^4\right)\left[\frac{\partial P(+0,y)}{\partial x}+\frac{\partial P(-0,y)}{\partial x}\right]+\nu_2\left[P(+0,y)-P(-0,y)\right]=0 \tag{1.10}$$
$$\frac{\partial P(+0,y)}{\partial x}=\frac{\partial P(-0,y)}{\partial x},\qquad y>0$$

Here the boundary-contact conditions are the following:

$$\lim_{x\to\pm 0}\frac{\partial P(x,0)}{\partial y}=0,\qquad \lim_{y\to 0}\frac{\partial P(\pm 0,y)}{\partial x}=0$$
$$\lim_{x\to\pm 0}\frac{\partial^2 P(x,0)}{\partial y\,\partial x}+\lim_{y\to 0}\frac{\partial^2 P(\pm 0,y)}{\partial x\,\partial y}=0 \tag{1.11}$$
$$D_1\lim_{x\to +0}\frac{\partial^3 P(x,0)}{\partial x^2\,\partial y}-D_1\lim_{x\to -0}\frac{\partial^3 P(x,0)}{\partial x^2\,\partial y}+D_2\lim_{y\to 0}\frac{\partial^3 P(\pm 0,y)}{\partial y^2\,\partial x}=0$$

Expanding the function $P(x,y)$ in even and odd parts in the variable x

$$P(x,y)={}^1/_2\left[P_+(x,y)+P_-(x,y)\right],\qquad P_\pm(x,y)=P(x,y)\pm P(-x,y)$$

let us separate the problem into two problems, each of which can be formulated just for positive values of x. For $P_+(x,y)$ we have $L_2=\partial/\partial x$ (L_1 is given by the first formula of (1.7)), and the two boundary-contact conditions are the first conditions (1.8) and

$$\lim_{x\to 0} \frac{\partial^2 P_+(x, 0)}{\partial x\, \partial y} = 0$$

The problem for $P_-(x, y)$ agrees with the problem in Example 1 if D_2 is replaced by $1/2\, D_2$ in the last condition of (1.8) and ν_2 by $2\nu_2$ in the second condition in (1.7).

Example 3. Diffraction by a cruciform intersection of infinite plates ($x = 0$, $y = 0$). The problem is reduced to four independent problems for a right angle by decomposition of the total field $P(x, y)$ into even and odd parts in each of the space variables. Their formulation is not difficult, and is omitted here in the interest of brevity.

Let us note that diffraction by a T connection of plates has already been examined [4, 5]; however, the exact solution of the problem taking account of the contact between the semi-infinite plate and the fluid is not contained in these papers.

2. General solution. A solution satisfying all the requirements of the problem except the boundary-contact conditions (1.6) is constructed below.

Let us represent the desired field $P(x, y)$ as the sum of three terms

$$P = P_0 + P^{(1)} + P^{(2)} \tag{2.1}$$

Here P_0 is the field of a point source whose coordinates (x_0, y_0) are in an infinite liquid medium. Let us use two modes of writing the field P_0

$$P_0(x, y) = \frac{1}{4\pi} \int_{-\infty}^{+\infty} \exp\,[i\lambda (x - x_0) - \sqrt{\lambda^2 - k^2}\,|y - y_0|]\, \frac{d\lambda}{\sqrt{\lambda^2 - k^2}} \tag{2.2}$$

$$P_0(x, y) = \frac{1}{4\pi} \int_{-\infty}^{+\infty} \exp\,[i\lambda (y - y_0) - \sqrt{\lambda^2 - k^2}\,|x - x_0|]\, \frac{d\lambda}{\sqrt{\lambda^2 - k^2}}$$

The terms $P^{(1)}$ and $P^{(2)}$ should separately satisfy the homogeneous Helmholtz equation, the requirement of continuity at the origin, and the principle of ultimate absorption. Let us subject them to the following boundary conditions:

$$L_1 P^{(1)}(x, 0) = -L_1 P_0(x, 0), \qquad L_2 P^{(1)}(0, y) = 0 \tag{2.3}$$
$$L_1 P^{(2)}(x, 0) = 0, \qquad L_2 P^{(2)}(0, y) = -L_2 P_0(0, y)$$

The boundary conditions (1.2) will thereby be satisfied automatically for P.

Let us limit ourselves to the problem for $P^{(1)}$ since finding $P^{(2)}$ is accomplished analogously. Let us seek $P^{(1)}$ in the form of an expansion in plane waves

$$P^{(1)}(x, y) = \frac{1}{4\pi} \int_{\Lambda_1} p_1(\lambda) \exp(-i\lambda x - \sqrt{\lambda^2 - k^2}\, y)\, d\lambda \tag{2.4}$$

The function $p_1(\lambda)$ and the contour Λ_1 are desired. As will be shown below, $p_1(\lambda)$ is some transcendental function, bifurcated at $\lambda = \pm k$ and with a certain set of poles. Assuming the estimate

$$p_1(\lambda) = O(\lambda^{-1-\varepsilon}) \quad (|\lambda| \to \infty, \quad \varepsilon > 0) \tag{2.5}$$

to hold in the neighborhood of the real axis, we satisfy the requirement for continuity at the origin.

In [3], where diffraction by two semi-infinite plates joined so that they are mutual extensions, analogous representation for the diffracted field takes part ; the integration is performed over the real axis. The difference between (2.4) and its corresponding representation of (2. 5) from [3] is determined by the requirement of the ultimate absorption principle. The possibility of deforming the contour of integration into the upper and lower half-planes from the real axis (as is done in [1]) must be available in the problem for a half-plane In the case considered here the negative values of the variable x do not belong to the domain where the solution is sought, and only the possibility of deforming the contour into the upper half-plane is needed. Hence, integration over some contour of more general shape can be used in place of integration over the real axis. Thus, if it is considered that the ends of the contour Λ_1 proceed along the real axis and only a certain number of poles $p_1(\lambda)$ is located between Λ_1 and the real axis for each of which

$$\operatorname{Im}\lambda > 0, \qquad \operatorname{Re}\sqrt{\lambda^2 - k^2} > 0$$

then (2, 4) will determine a function satisfying the ultimate absorption principle as before. Let us henceforth assume that the desired contour possesses the properties listed. The specific selection of the contour will be accomplished below.

Using the boundary conditions, we arrive at the following integral equation for the desired functions :

$$\frac{1}{4\pi}\int_{\Lambda_1} l_1(\lambda)\, p_1(\lambda)\exp(i\lambda x)\, d\lambda =$$

$$-\frac{1}{4\pi}\int_{-\infty}^{+\infty} l_1^\circ(\lambda)\exp\left(i\lambda(x-x_0)-\sqrt{\lambda^2-k^2}y_0\right)\frac{d\lambda}{\sqrt{\lambda^2-k^2}} \tag{2.6}$$

$$\int_{\Lambda_1} m_2(\lambda)\, p_1(\lambda)\exp\left(-\sqrt{\lambda^2-k^2}y\right)d\lambda = 0 \tag{2.7}$$

$$l_\beta^\circ(\lambda) = m_{\alpha 1}(\lambda^2)\sqrt{\lambda^2-k^2} + m_{\alpha 2}(\lambda^2) \tag{2.8}$$

The algebraic function $l_1^\circ(\lambda)$ has no poles, hence, integration over the real axis can be replaced by integration over Λ_1 in the right side of (2. 6). We consequently have

$$\int_{\Lambda_1}\left[l_1(\lambda)\, p_1(\lambda) + \frac{1}{\sqrt{\lambda^2-k^2}} l_1^\circ(\lambda)\exp\left(-i\lambda x_0-\sqrt{\lambda^2-k^2}y_0\right)\right]\exp(i\lambda x)\, d\lambda = 0 \tag{2.9}$$

Let us set

$$l_1(\lambda)\, p_1(\lambda) + \frac{l_1^\circ(\lambda)}{\sqrt{\lambda^2-k^2}}\exp\left(-i\lambda x_0-\sqrt{\lambda^2-k^2}y_0\right) = \Phi^+(\lambda) \tag{2.10}$$

to satisfy (2. 9), where $\Phi^+(\lambda)$ is a function analytic above the contour Λ_1.

Equation (2. 7) will be satisfied if the integrand is assumed odd

$$m_2(\lambda)\, p_1(\lambda) = -m_2(-\lambda)\, p_1(-\lambda) \tag{2.11}$$

and the contour of integration is assumed symmetric relative to the origin. In order to satisfy the ultimate absorption principle, let us assume that there are no singularities of $p_1(\lambda)$ above the contour of integration in the lower half-plane. The selection of the contour Λ_1 in application to the case examined in Example 1 is shown in Fig. 1. The part of the Λ_1 contour located on the main sheet of the Riemann surface is mapped by the solid line, and the slits are shown by the dashes. The roots of the $l_1(\lambda)$ and $m_2(\lambda)$ are denoted respectively by $\pm\lambda_{1n}$ and μ_{2n} ($n = 0, 1, \ldots 4$), the roots of $l_1(\lambda)$ are on the second sheet mapped by the circles. Eliminating $p_1(\lambda)$ from (2.10) and (2.11), we arrive at a Riemann boundary value problem to find the piecewise-analytic function $\Phi^+(\pm\lambda)$ by means of a linear relationship connecting its limit values on both sides of the contour Λ_1

$$m_2(\lambda)\,\Phi^+(\lambda) + m_2(-\lambda)\,\Phi^+(-\lambda) = \psi_1(\lambda), \quad (\lambda)\in\Lambda_1 \tag{2.12}$$

$$\psi_1(\lambda) = \frac{l_1^\circ(\lambda)}{\sqrt{\lambda^2-k^2}}\exp\left(-\sqrt{\lambda^2-k^2}\,y_0\right)\left[m_2(\lambda)\exp(-i\lambda x_0) + m_2(-\lambda)\exp(i\lambda x_0)\right] \tag{2.13}$$

By using a Cauchy type integral we represent $\psi_1(\lambda)$ as the difference between the

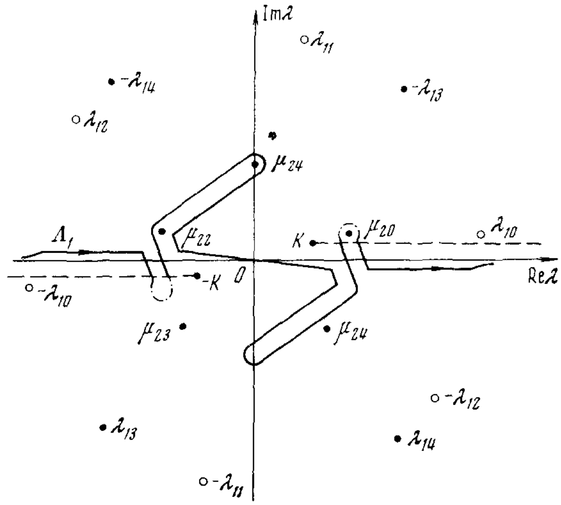

Fig. 1

limit values above (Ψ_1^+) and below (Ψ_1^-) the contour Λ_1 for the piecewise-analytic function Ψ_1

$$\psi_1(\lambda) = \Psi_1^+(\lambda) - \Psi_1^-(\lambda), \qquad \lambda\in\Lambda_1 \tag{2.14}$$

$$\Psi_1(\lambda) = \frac{1}{2\pi i}\int_{\Lambda_1}\frac{\psi_1(\tau)\,d\tau}{\tau-\lambda} \tag{2.15}$$

It is easy to see that the formula

$$\Psi_1^-(-\lambda) = -\Psi_1^+(\lambda) \tag{2.16}$$

holds everywhere on the main sheet. On the basis of (2.12), (2.14), (2.16), we have

$$m_2(\lambda)\,\Phi^+(\lambda) - \Psi_1^+(\lambda) = -m_2(-\lambda)\,\Phi^+(-\lambda) + \Psi_1^+(-\lambda),\ \lambda\in\Lambda_1 \tag{2.17}$$

from which the left and right sides of (2.17) yield some function analytic and odd in the whole complex λ plane in conformity with the Riemann theorem on analytic continuation across a contour. The estimate (2.5) will be satisfied if this function is selected as a polynomial of degree $2N-1$, where N is defined by (1.5). Therefore

$$m_2(\lambda)\,\Phi^+(\lambda) - \Psi_1^+(\lambda) = \lambda q_{N-1}^{(1)}(\lambda^2) \tag{2.18}$$

from which

$$\Phi^+(\lambda) = \frac{\Psi_1^+(\lambda) + \lambda q_{N-1}^{(1)}(\lambda^2)}{m_2(\lambda)} \tag{2.19}$$

$$p_1(\lambda) = -\frac{l_1^\circ(\lambda)}{l_1(\lambda)} \frac{\exp(-i\lambda x_0 - \sqrt{\lambda^2 - k^2} y_0)}{\sqrt{\lambda^2 - k^2}} + \frac{\Psi_1^+(\lambda) + \lambda q_{N-1}^{(l)}(\lambda^2)}{l_1(\lambda) m_2(\lambda)} \tag{2.20}$$

The function $p_1(\lambda)$ has a pole at the roots of the polynomial $m_2(\lambda)$ and the algebraic function $l_1(\lambda)$. Let us recall that none of the roots of $m_2(\lambda)$ is a root of $l_1(\lambda)$ (property (1)). In this case the contour Λ_1 is selected as follows:

1. The contour Λ_1 is symmetric relative to the origin and coincides at infinity with the real axis.

2. All the roots of $l_1(\lambda)$ in the upper half-plane are above Λ_1.

3. All the roots of $m_2(\lambda)$ are below Λ_1, where they are bypassed (taking account of the bifurcation of $p_1(\lambda)$ at the point $\lambda = k$) in such a way that $\mathrm{Re}\sqrt{\lambda^2 - k^2} > 0$ at each.

We consequently have the following expression for the field $P^{(1)}$:

$$P^{(1)} = P_1 + P_1' + Q_1$$

$$P_1 = -\frac{1}{4\pi} \int_{\Lambda_1} \frac{l_1^\circ(\lambda)}{l_1(\lambda)} \exp(i\lambda(x - x_0) - \sqrt{\lambda^2 - k^2}(y + y_0)) \frac{d\lambda}{\sqrt{\lambda^2 - k^2}}$$

$$P_1' = \frac{1}{4\pi} \int_{\Lambda_1} \frac{\Psi_1^+(\lambda)}{l_1(\lambda) m_2(\lambda)} \exp(i\lambda x - \sqrt{\lambda^2 - k^2} y)\, d\lambda \tag{2.21}$$

$$Q_1 = \frac{1}{4\pi} \int_{\Lambda_1} \frac{\lambda q_{N-1}^{(1)}(\lambda^2)}{l_1(\lambda) m_2(\lambda)} \exp(i\lambda x - \sqrt{\lambda^2 - k^2} y)\, d\lambda$$

and $P^{(2)}$ is found analogously. The total expression for $P^{(2)}$ is obtained from (2.21) by a cyclic replacement of the subscripts 1 and 2 and the variables x and y. Hence

$$Q_2(x, y) = \frac{1}{4\pi} \int_{\Lambda_2} \frac{\lambda q_{N-1}^{(2)}(\lambda^2)}{l_2(\lambda) m_1(\lambda)} \exp(i\lambda y - \sqrt{\lambda^2 - k^2} x)\, d\lambda \tag{2.22}$$

agrees with $Q_1(x, y)$ to the accuracy of the notation of the polynomial coefficients (it is sufficient to replace the variable of integration λ in (2.22) by $i\sqrt{\lambda^2 - k^2}$). The final expression for the total diffraction field is

$$P = P_0 + P_1 + P_2 + P_3 + Q \tag{2.23}$$

$$P_2(x, y) = -\frac{1}{4\pi} \int_{\Lambda_2} \frac{l_2^\circ(\lambda)}{l_1^\circ(\lambda)} \exp(i\lambda(y - y_0) - \sqrt{\lambda^2 - k^2}(x + x_0)) \frac{d\lambda}{\sqrt{\lambda^2 - k^2}}$$

$$Q(x, y) = \frac{1}{4\pi} \int_{\Lambda_1} \frac{\lambda q_{N-1}(\lambda^2)}{l_1(\lambda) m_2(\lambda)} \exp(i\lambda x - \sqrt{\lambda^2 - k^2} y)\, d\lambda = \frac{1}{4\pi} \int_{\Lambda_2} \frac{\lambda q_{N-1}(k^2 - \lambda^2)}{l_2(\lambda) m_1(\lambda)} \times \exp(i\lambda y - \sqrt{\lambda^2 - k^2} x)\, d\lambda$$

where P_0 and P_1 are given by (2.2) and (2.21), and P_3 is the sum of P_1' and P_2'. After some manipulation we have

$$P_3(x,\ y) = \frac{1}{4\pi} \int\limits_{\Lambda} \frac{l_1^\circ(\lambda)\, m_2(-\lambda)}{l_1(\lambda)\, m_2(\lambda)} \exp\left(i\lambda(x+x_0) - \sqrt{\lambda^2-k^2}\,(y+y_0)\right) \frac{d\lambda}{\sqrt{\lambda^2-k^2}}$$

Here P_1 has the meaning of a wave reflected from the boundary $(x>0,\ y=0)$ if it is continued to infinity for negative x. Under the Dirichlet or Neumann boundary conditions, P_1 goes over into the field of an imaginary source at the point $(x_0,\ -y_0)$. Under the Dirichlet or Neumann boundary conditions P_2 goes over analogously into the field of an imaginary source at the point $(-x_0,\ y_0)$, and P_3 - at the point $(-x_0,\ -y_0)$. The sum $P_0+P_1+P_2+P_3$ satisfies all the constraints of the problem posed in Sect.1, and has continuous derivatives of order N_1+N_2-1 inclusive, at the origin. Here Q satisfies the homogeneous Helmholtz equation, the homogeneous boundary conditions (1.2), and has discontinuities in the second derivatives of the field at the origin. Let us note that, in contrast to the case of longitudinal junctions of plates, the solution obtained does not contain the products of factorization of the functions $l_{1,2}(\lambda)$ and is simpler and more convenient for investigation. The expression obtained for Q retains its form independently of the assignment of the incident field. As the field source changes, only the numerical values of the constants of the polynomial q_{N-1} vary.

3. Boundary-contact conditions. Regularization of the divergent integrals originating in the formal application of the boundary-contact operators is indicated below, and the algebraic system to find the constants in Q is also written down (see Example 1 in Sect.1).

The application of the boundary-contact operators on P_0, P_1 and P_2 encounters no difficulties since the exponential decrease at infinity is conserved in the integrands of the integrals obtained here. The integrand on P_3 behaves as $|\lambda|^{N_1-N_2-1}$ at infinity for $x=0,\ y=0$. If it is assumed that the differential order of $R_{\alpha\beta}$ does not exceed N_1+N_2-1, then application of the boundary-contact operators will not result in the generation of divergent integrals. Regularization of divergent integrals of the type $R_{\alpha\beta}\ Q\ (0,\ 0)$ can be accomplished by the example described in [3]. Regularization of such integrals by this method is possible if

$$r_{\alpha\beta}(\lambda)\, l_\alpha^\circ(\lambda) - r_{\alpha\beta}^\circ(\lambda)\, l_\alpha(\lambda) = O(|\lambda|^{N_\alpha}) \tag{3.1}$$

$$r_{\alpha\beta}(\lambda) = -s_{\alpha\beta1}(\lambda)\sqrt{\lambda^2-k^2} + s_{\alpha\beta2}(\lambda)$$

$$r_{\alpha\beta}^\circ(\lambda) = s_{\alpha\beta1}(\lambda)\sqrt{\lambda^2-k^2} + s_{\alpha\beta2}(\lambda)$$

is satisfied.

Let us return to Example 1 from Sect.1. The general solution here contains four arbitrary constants, and the field Q can be written as

$$Q = \frac{1}{4\pi} \int\limits_{\Lambda_1} \frac{\lambda(a\lambda^6 + b\lambda^4 + c\lambda^2 + d)}{l_1(\lambda)\, m_2(\lambda)} \exp\left(i\lambda x - \sqrt{\lambda^2-k^2}\, y\right) d\lambda =$$

$$\frac{1}{4\pi} \int\limits_{\Lambda_2} \frac{\lambda[a(k^2-\lambda^2)^3 + b(k^2-\lambda^2) + c(k^2-\lambda^2) + d]}{l_2(\lambda)\, m_1(\lambda)} \exp\left(i\lambda y - \sqrt{\lambda^2-k^2}\, x\right) d\lambda$$

$$l_\alpha(\lambda) = -(\lambda^4 - k_\alpha^4)\sqrt{\lambda^2-k^2} + \nu_\alpha,\quad m_\alpha = i\lambda[(\lambda^2-k^2)^2 - k_\alpha^4] + \nu_\alpha$$

Using the boundary-contact conditions (1.8), we obtain the following system to find $a,\ b,\ c,\ d$:

$$aT_{1,7} + bT_{1,5} + cT_{1,3} + dT_{1,1} = A_1$$

$$a(k^6T_{2,1} - 3k^4T_{2,3} + 3k^2T_{2,5} - T_{2,7}) + b(k^4T_{2,1} - 2k^2T_{2,3} + T_{2,5}) + c(k^2T_{2,1} - T_{2,3}) + dT_{2,1} = A_2 \tag{3.2}$$

$$a\,(T_{1,8} - k^6T_{2,2} + 3k^4T_{2,4} - 3k^2T_{2,6} + T_{2,8}) + b\,(T_{1,6} - k^4T_{2,2} + 2k^2T_{2,4} - T_{2,6}) + c\,(T_{1,4} - k^2T_{2,2} + T_{2,4}) + d\,(T_{1,2} - T_{2,2}) = A_3$$

$$a\,[D_1T_{1,9} + D_2\,(k^6T_{2,3} - 3k^4T_{2,5} + 3k^2T_{2,7} - T_{2,9})] + b\,[D_1T_{1,7} + D_2\,(k^4T_{2,3} - 2k^2T_{2,5} + T_{2,7})] + c\,[D_1T_{1,5} + D_2\,(k^2T_{2,3} - T_{2,5})] + d\,(D_1T_{1,3} + D_2T_{2,3}) = A_4$$

$$T_{1,s} = \frac{\nu_1}{4\pi}\int_{-\infty}^{\infty}\frac{\lambda^s\sqrt{\lambda^2-k^2}\,d\lambda}{m_2(\lambda)\,l_1(\lambda)\,l_1^\circ(\lambda)} + \frac{i}{2}\sum_{l_1l_1^\circ=0}\operatorname{Res}\frac{\lambda^s(\lambda^2-k^2)(\lambda^4-k_1^4)}{m_2(\lambda)\,l_1(\lambda)\,l_1^\circ(\lambda)} - \frac{i\nu_1}{2}\sum_{m_2=0}\operatorname{Res}\frac{\lambda^s\sqrt{\lambda^2-k^2}}{m_2(\lambda)\,l_1(\lambda)\,l_1^\circ(\lambda)} \qquad (3.3)$$

$$\operatorname{Im}\lambda > 0,\quad \operatorname{Re}\sqrt{\lambda^2-k^2} > 0,\quad \operatorname{Im}\lambda > 0,\quad \operatorname{Re}\sqrt{\lambda^2-k^2} > 0$$

The formula for $T_{2,s}$ is obtained from (3.3) by a cyclic replacement of the subscripts 1 and 2, where A_β ($\beta = 1, 2, 3, 4$) are numbers obtained because of using the operators in the right sides of (1.8) on the expression $P_0 + P_1 + P_2 + P_3$ (they are not written down in the interests of brevity).

Let us note that the form of the left sides of (3.2) is independent of the nature of the incident wave, but is determined by the properties of the model itself. Only numerical values of the right sides are changed by a different selection of the field source. The integrals $T_{\alpha s}$ are similar to the integrals in problems of acoustic diffraction by point disturbances of the elastic properties of plates [6, 7] and can be studied by analogous methods.

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